Intermediate Microeconomics

Lecture 1

Hessian Matrix

$$H = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} & \cdots & \frac{\partial^2}{\partial x_n \partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2}{\partial x_1 \partial x_n} & \cdots & \frac{\partial^2}{\partial x_n^2} \end{bmatrix}$$

Alternating sign of the determinant of successive matrices in the hessian implies a maximum

All positive determinants of successive matrices in the hessian implies the existence of a minimum

Lecture 2

Demand functions describe consumer behavior

Residual demand is the consumer demand a single firm faces

Consumption bundle (x_1, x_2) contains x_1 units of commodity 1 and x_2 units of commodity 2

Consumers have preferences between consumption bundles

We can represent consumer preferences by utility functions

Our goal is then to maximize utility

Two-variable unconstrained optimization (Interior)

$$\max_{x_1 x_2} u(x_1, x_2) \implies \frac{\partial u(x_1^*, x_2^*)}{\partial x_1} = \frac{\partial u(x_1^*, x_2^*)}{\partial x_2} = 0$$ ← Necessary conditions

Hessian Matrix : $\begin{bmatrix} \frac{\partial^2 u}{\partial x_1^2} & \frac{\partial^2 u}{\partial x_1 \partial x_2} \\ \frac{\partial^2 u}{\partial x_1 \partial x_2} & \frac{\partial^2 u}{\partial x_2^2} \end{bmatrix}$ ← Sufficient conditions

For a maximum :

$$\frac{\partial^2 u}{\partial x_1^2} < 0 \quad \text{and} \quad |H| = \frac{\partial^2 u}{\partial x_1^2} \cdot \frac{\partial^2 u}{\partial x_2^2} - \left(\frac{\partial u}{\partial x_1 \partial x_2}\right)^2 > 0$$

$$\begin{bmatrix} \frac{\partial^2 u}{\partial x_1^2} & \frac{\partial^2 u}{\partial x_1 \partial x_2} \\ \frac{\partial^2 u}{\partial x_1 \partial x_2} & \frac{\partial^2 u}{\partial x_2^2} \end{bmatrix}$$

For a minimum :

$$\frac{\partial^2 u}{\partial x_1^2} > 0 \quad \text{and} \quad |H| = \frac{\partial^2 u}{\partial x_1^2} \cdot \frac{\partial^2 u}{\partial x_2^2} - \left(\frac{\partial u}{\partial x_1 \partial x_2}\right)^2 > 0$$

$$\begin{bmatrix} \frac{\partial^2 u}{\partial x_1^2} & \frac{\partial^2 u}{\partial x_1 \partial x_2} \\ \frac{\partial^2 u}{\partial x_1 \partial x_2} & \frac{\partial^2 u}{\partial x_2^2} \end{bmatrix}$$

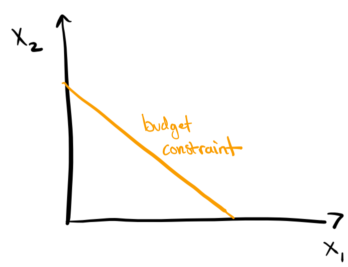
Constrained Optimization

$$\max_{x_1, x_2} u(x_1, x_2) \quad \text{subject to} \quad g(x_1, x_2) = 0$$

Budget constraint

A consumption bundle is said to be affordable for prices p_1 and p_2

$$\underbrace{p_1 x_1 + p_2 x_2}_{\text{Expenditure}} \leq \underset{\uparrow \text{ income}}{I}$$



Lagrange Multipliers

Suppose we want to optimize our utility subject to the budget constraint

constraint: $P_1X_1 + P_2X_2 - I = 0$

changing this sign changes the final sign of λ

$$L(X_1, X_2, \lambda) = U(X_1, X_2) + \lambda[P_1X_1 + P_2X_2 - I]$$

Lagrangian function

$$= X_1 \cdot X_2 + \lambda[P_1X_1 + P_2X_2 - I]$$

$$\frac{\partial L}{\partial X_1} = X_2 + \lambda P_1 = 0 \Rightarrow \lambda = \frac{-X_2}{P_1}$$

$$\frac{\partial L}{\partial X_2} = X_1 + \lambda P_2 = 0 \Rightarrow \lambda = -X_1/P_2$$

$$\Rightarrow \frac{-X_2}{P_1} = -\frac{X_1}{P_2} \Rightarrow X_2 = \frac{P_1}{P_2}X_1$$

$$\frac{\partial L}{\partial \lambda} = P_1X_1 + P_2X_2 - I = 0$$

budget restraint

Plugging in our expression for X_2 into the budget constraint we find

$$P_1X_1 + P_1X_1 = I$$

$$X_1^* = \frac{I}{2P_1}, \quad X_2^* = \frac{I}{2P_2}$$

$$\lambda^* = \frac{-I}{2P_1P_2}$$

$$V(X_1^*, X_2^*) = X_1^* \cdot X_2^*$$

maximum utility

$$= \frac{I^2}{4P_1P_2}$$

$$\frac{\partial V}{\partial I} = \frac{I}{2P_1P_2} = |\lambda|$$

Generalized Lagrange Optimization

$$L = U(X_1, X_2) + \lambda[X_1P_1 + P_2X_2 - I]$$

$$\frac{\partial L}{\partial X_1} = \frac{\partial U}{\partial X_1} + \lambda P_1 = 0 \Rightarrow \lambda = -\frac{\frac{\partial U}{\partial X_1}}{P_1}$$

marginal benefit of X_1

marginal cost of X_1

$$\frac{\partial L}{\partial X_2} = \frac{\partial U}{\partial X_2} + \lambda P_2 = 0 \Rightarrow \lambda = -\frac{\frac{\partial U}{\partial X_2}}{P_2}$$

At max all goods have the same $\frac{\text{marginal benefit}}{\text{marginal cost}}$

Lecture 3

$$\max_{X_1, X_2} U(X_1, X_2) = X_1 \cdot X_2^2 \quad \text{s.t.} \quad P_1X_1 + P_2X_2 = I$$

$$P_1X_1 + P_2X_2 - I = 0$$

$$L(X_1, X_2, \lambda) = X_1 X_2^2 + \lambda[P_1X_1 + P_2X_2 - I]$$

$$\frac{\partial L}{\partial X_1} = X_2^2 + \lambda P_1 = 0$$

$$\frac{\partial L}{X_2} = 2X_1X_2 + \lambda P_2$$

$$\Rightarrow \frac{X_2^2}{P_1} = \frac{2X_1X_2}{P_2} \Rightarrow X_2 = \frac{P_1}{P_2} \cdot 2X_1 \quad \text{B}$$

$$\frac{\partial L}{\partial \lambda} = P_1X_1 + P_2X_2 - I = 0 \quad \text{A}$$

(A) $P_1 x_1 + P_2 \left(\frac{P_1}{P_2} \cdot 2x_1 \right) = I \Rightarrow 3P_1 x_1 = I$

$\Rightarrow x_1^* = \frac{I}{3P_1}$ $\quad \uparrow P_1 \rightarrow \downarrow x_1^*$

$\quad \downarrow P_1 \rightarrow \uparrow x_1^*$

(B) $x_2^* = \frac{P_1}{P_2} \cdot 2x_1^* = \frac{P_1}{P_2} \cdot 2\frac{I}{3P_1} \Rightarrow x_2^* = \frac{2I}{3P_2}$

(C) $\Rightarrow \lambda^*$

$P_1 \cdot x_1^* = P_1 \cdot \frac{I}{3 \cdot P_1} = \frac{I}{3}$

$P_2 \cdot x_2^* = P_2 \cdot \frac{2I}{3 \cdot P_2} = \frac{2I}{3}$

Generally,

$U(x_1, x_2) = x_1^a x_2^b$

$\frac{a}{a+b}$ $\quad \frac{b}{a+b}$ ← fraction of income

A function is said to be homogeneous of degree k if

$f(tx_1, tx_2, \ldots tx_n) = t^k f(x_1, x_2, x_3, \ldots x_n)$

Second-Order Conditions w/ Lagrange Optimization

Consider the previous example

$L(x_1, x_2, \lambda) = x_1 x_2^2 + \lambda [P_1 x_1 + P_2 x_2 - I]$

$L_1 = \frac{\partial L}{\partial x_1} = x_2^2 + \lambda P_1 = 0$

$L_2 = \frac{\partial L}{x_2} = 2x_1 x_2 + \lambda P_2$

$L_\lambda = \frac{\partial L}{\partial \lambda} = P_1 x_1 + P_2 x_2 - I = 0$

Hessian Matrix

$$B = \begin{array}{c|ccc|} & \lambda & x_1 & x_2 \\ \lambda & L_{\lambda\lambda} & L_{\lambda 1} & L_{\lambda 2} \\ x_1 & L_{1\lambda} & L_{11} & L_{12} \\ x_2 & L_{2\lambda} & L_{21} & L_{22} \end{array} \longrightarrow \begin{bmatrix} 0 & P_1 & P_2 \\ P_1 & 0 & 2x_2 \\ P_2 & 2x_2 & 2x_1 \end{bmatrix}$$

Conditions for a maximum

$\begin{vmatrix} L_{\lambda\lambda} & L_{\lambda 1} \\ L_{1\lambda} & L_{11} \end{vmatrix} < 0$

and

$|B| > 0$

Conditions for a minimum

$\begin{vmatrix} L_{\lambda\lambda} & L_{\lambda 1} \\ L_{1\lambda} & L_{11} \end{vmatrix} < 0$

and

$|B| < 0$

Lecture 4

Consumer choice is based on preferences, budget constraint and optimal choice

We denote consumption preferences $(x_1, x_2) \succeq (y_1, y_2)$ to say that consumption bundle x is weakly preferred to y

We compare consumption bundles on the basis of utility

More preferred bundles have higher utility

Axioms of Rational Choice

① Completeness : Any two bundles can be compared

$(x_1, x_2) \succeq (y_1, y_2) \rightarrow$ weakly preferred to y

$(y_1, y_2) \succeq (x_1, x_2) \rightarrow$ weakly preferred to x

$(x_1, x_2) \sim (y_1, y_2) \rightarrow$ indifferent

② Transitivity : If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$

③ Continuity : If $(x_1, x_2) \succeq (y_1, y_2)$ then small deviations from (x_1, x_2) are preferred to (y_1, y_2)

Utility functions are used to rank consumption bundles

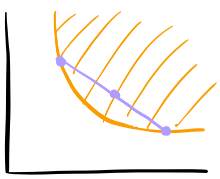
Indifference curves are consumption bundles of equal utility

Upper contour set is the set of consumption bundles with utilities greater than some given level

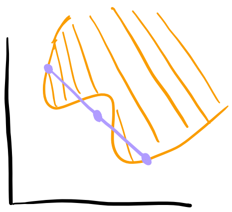
Monotonic preferences always increase utility as the quantity of the commodity increases

Quasiconcavity refers to the idea that consumers prefer balanced "diverse" bundles of goods to extreme one

Formally, given two bundles on an indifference curve, the average of the two points exists in the upper contour set



Valid Utility



Invalid Utility

A collection S of bundles is convex if any line between two bundles in S lies in S

U is said to be quasiconcave if all of its upper contour sets are convex

We can verify that U is quasiconcave by proving that ICs are concave

Suppose $U(x_1, x_2) = x_1 x_2$

Our IC is when $x_1 x_2 = \bar{u}$

$$x_2 = g(x_1) = \frac{\bar{u}}{x_1}$$

$$\frac{dg(x_1)}{dx_1} = \frac{-\bar{u}}{x_1^2} < 0$$

$$\frac{d^2 g(x_1)}{dx_1^2} = \frac{2\bar{u}}{x_1^3} > 0 \leftarrow \text{Implies convexity}$$

When U is quasiconcave and the constraint is linear we can ignore second-order conditions when finding constrained maximum

Marginal rate of substitution is the slope of the indifference curve

Rate an individual would be willing to trade x_2 for x_1 while maintaining utility

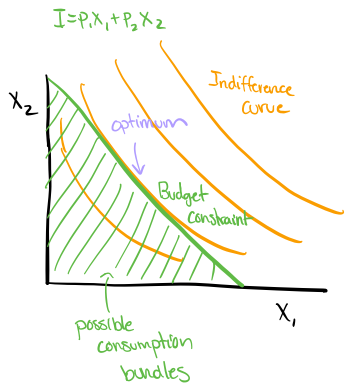
Diminishing marginal rate of substitution refers to the phenomena that as x_1 increases the more you are willing to sacrifice x_1 for x_2

Solving Marginal Rate of Substitution

Given $U(x_1, x_2)$ we want $dU = 0$

$$dU = 0 = \frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2$$

$$\frac{dx_2}{dx_1} = -\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = -\frac{\text{Marginal Utility 1}}{\text{Marginal Utility 2}} = \text{MRS}$$



$MRS|_{optimum} = P_1/P_2$

Lagrange optimization finds point of tangency

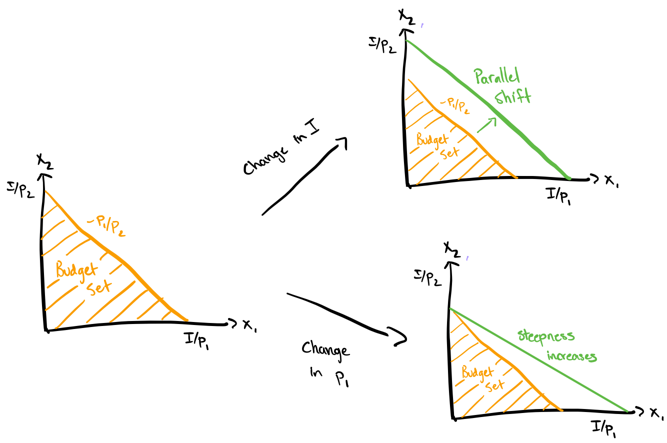
Individual consumer allighs with market pricing

Lecture 5

We define a budget set to be the set of bundles affordable for given prices and incomes

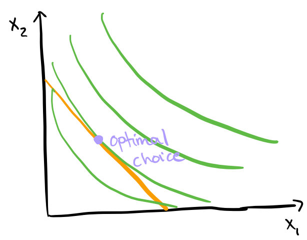
For a given budget constraint $I = p_1x_1 + p_2x_2$ we can solve $x_2 = -\frac{P_1}{P_2}x_1 + I/P_2$

This slope $-P_1/P_2$ is the rate at which the market is willing to substitute x_2 for x_1



Optimal consumption occurs when the IC is tangent to the budget line

consumers marginal rate of substitution is the price ratio



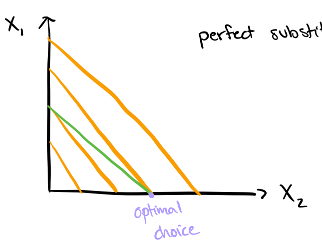
Homothetic Utility Functions : utility functions that are increasing transformations of one another give rise to the same demand

V is said to be an increasing transformation of u if $V = f(u)$ for some function $f(y)$ with $f' > 0$

Perfect substitutes : Consumer is willing to substitute one good for the other at a constant rate

$u(x_1, x_2) = ax_1 + bx_2$ for $a, b > 0$

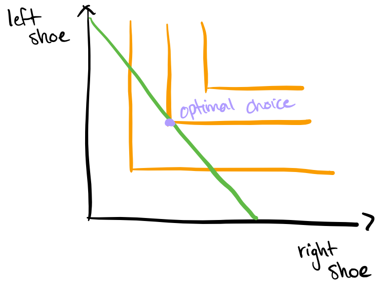
$MRS = \frac{a}{b}$



perfect substitutes yield corner solutions

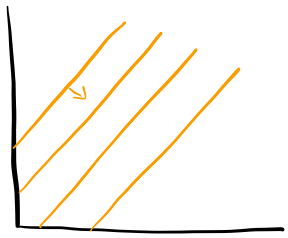
Perfect complements : L-shaped indifference curves. Consumer wants to consume goods in fixed proportions to eachother

$u(x_1, x_2) = \min\{ax_1, bx_2\}$ where $a, b > 0$



MRS is 0 or infinity

Bads : Indifference curves with negative slopes



Substitutes vs. Complements

- x_2 is a substitute for x_1 if $\frac{dx_2^*}{dp_1} > 0$ ← consume more x_2 if p_1 increases
- x_2 is a complement for x_1 if $\frac{dx_2^*}{dp_1} < 0$ ← consume less x_2 if p_1 increases

Lecture 6

Price Elasticity of Demand

How does a change in p_1 effect total expenditure on x_1?

Suppose we have a demand function $x^*(p_1)$

Total expenditure : $p_1 x_1^*(p_1)$

We want to consider
$$\frac{dp_1x_1^*(p_1)}{dp_1} = \underbrace{x_1^*(p_1)}_{\text{direct effect}} + \underbrace{x_1^{*\prime}(p_1)\,p_1}_{\text{indirect effect (behavioural)}} = x_1^*\left[1 + \frac{p_1}{x_1^*}\frac{dx^*}{dp_1}\right] = x_1^*\left[1 - \varepsilon_p\right]$$

$$\underbrace{\varepsilon_p = -\frac{dx_1^*}{dp_1}\frac{p_1}{x_1^*}}_{\text{defined s.t. } \varepsilon_p > 0}$$

If price elasticity = 1 (Unit Elastic)

$$\varepsilon_p = 1 \rightarrow \frac{dp_1x_1^*(p_1)}{dp_1} = 0$$ ← unit elastic

Change in price is equal and opposite to demand

Suppose $\varepsilon_p > 1$ (Elastic)

$$\frac{dp_1x_1^*(p_1)}{dp_1} < 0$$ decrease in expenditure

demand is very sensitive to price

Suppose $\varepsilon_p < 1$ (Inelastic)

$$\frac{dp_1x_1^*(p_1)}{dp_1} > 0$$ increase in expenditure

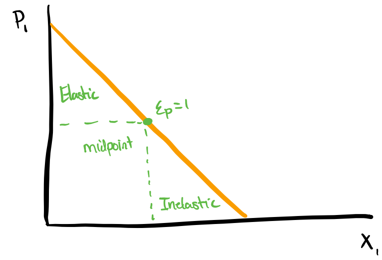
demand isn't that sensitive to price

Consider the cobb-douglas utility function $u(x_1,x_2) = x_1x_2^2$ => Demand : $x_1^* = \frac{I}{3p_1}$

$$\varepsilon_p = -\frac{dx_1^*}{dp_1}\frac{p_1}{x_1} = -\left(-\frac{I}{3p_1^2}\right)\cdot\frac{p_1}{I/3p_1} = 1$$

Income elasticity → $$\varepsilon_I = \frac{dx_1^*}{dI}\frac{I}{x_1^*} = \frac{1}{3p_1}\cdot\frac{I}{I/3p_1} = 1$$

Consider Linear demand



Changes in income shift demand in a parallel fashion

Marginal rate of substitution is constant

Normal good: The demand for X_1 increases as income increases

$$\frac{\partial x_1^*}{\partial I} > 0$$

Inferior good: the demand for x decreases as income increases

$$\frac{\partial x^*}{\partial I} < 0$$

ex. junk food, second-hand clothing etc.

Changes in price change the slope of the budget line

MRS is subject to change

If a good is normal, substitution and income effect reinforce eachother

Substitution effect: when $P_x \downarrow$ $x^* \uparrow$

As individual stays on original IC

Income effect: when $P_x \downarrow$ $x^* \uparrow$

As individual shifts to higher IC

If a good is inferior, substitution and income effects work in opposing directions

Substitution effect: when $P_x \downarrow$ $x^* \uparrow$

Income effect: when $P_x \downarrow$ $x^* \downarrow$

A giffen good is a good whose demand is increasing in its price

Income effect outweighs substitution effects

Lower prices → lower consumption

Lecture 7

Malthus Theory of Fertility: fertility increases when income increases

Normal Good: $\frac{\partial x^*}{\partial I} > 0$

Inferior Good: $\frac{\partial x^*}{\partial I} < 0$

When $P_x \downarrow$

finds new slope

Substitution effect: $x \uparrow$

movement along IC

Income effect: $x \uparrow$

movement to a higher IC

Normal good

jumps to correct IC

Substitution and income effect act in opposing directions with inferior goods

Giffen good is one whose demand increases as price increases

Income effect outweighs substitution effect

Dual Problem: Reach minimum utility while minimizing expenditure

Expenditure is just enough to reach $\bar{u}$

$$\min \; p_1x_1 + p_2x_2 \quad \text{s.t.} \quad x_1x_2^2 = \bar{u}$$

Apply lagrangian multiplier optimization

$$L(x_1, x_2, \lambda) = p_1x_1 + p_2x_2 + \lambda(x_1x_2^2 - \bar{u})$$

$$\frac{\partial L}{\partial x_1} = P_1 + \lambda x_2^2 = 0 \quad \Rightarrow \quad \lambda = -P_1/x_2^2$$

$$\frac{\partial L}{\partial x_2} = P_2 + 2\lambda x_1 x_2 = 0 \quad \Rightarrow \quad \lambda = -P_2/2x_1x_2$$

$$\Rightarrow \frac{-P_1}{x_2^2} = \frac{-P_2}{2x_1x_2} \rightarrow 2x_1P_1 = P_2x_2 \qquad \frac{P_1}{P_2} = \frac{x_2}{2x_1}$$

price ratio

Marginal rate of Substitution

$$\frac{\partial L}{\partial \lambda} = x_1x_2^2 - \bar{u} = 0$$

Plugging into the constraint

$$x_2 = \frac{2P_1}{P_2}x_1$$

$$x_1x_2^2 = \bar{u} \Rightarrow x_1\left(\frac{2P_1}{P_2}x_1\right)^2 = \bar{u} \Rightarrow x_1^3 = \frac{P_2^2\bar{u}}{4P_1^2} \Rightarrow x_1 = \left(\frac{P_2^2\bar{u}}{4P_1^2}\right)^{\frac{1}{3}}$$

compensated demand

Minimum expenditure

$$E = P_1 \bar{u}^{-1/3}\left(\frac{P_2}{2P_1}\right)^{2/3} + P_2\bar{u}^{-1/3}\left(\frac{2P_1}{P_2}\right)^{1/3}$$

Shepard's Lemma

Compensated demand for a good can be found from the expenditure function by differentiation w.r.t good's price

$$\frac{\partial E(P_1, P_2, \bar{u})}{\partial P_1} = x_1^c(P_1, P_2, \bar{u})$$

Uncompensated Marshallian Demand (Max utility given budget constraint)

Income and substitution effect captured in demand curve

Compensated Hicksian Demand (min expenditure for given utility)

Only captures substitution effect

Compensated demand is less responsive to changes than uncompensated demand

Hicksian demand ignores income effect

$$x_1^c(P_1, P_2, \bar{u}) = x_1^*[P_1, P_2, E(P_1, P_2, \bar{u})]$$

$$\frac{\partial x_1^c}{\partial P_1} = \frac{\partial x_1^*}{\partial P_1} + \frac{\partial x_1^*}{\partial E}\cdot\frac{\partial E}{\partial P_1}$$

$$\frac{\partial x_1^*}{\partial P_1} = \underbrace{\frac{\partial x_1^c}{\partial P_1}}_{\text{Substitution effect}} - \underbrace{\frac{\partial x_1^*}{\partial E}\cdot\frac{\partial E}{\partial P_1}}_{\text{Income effect}}$$

Income Effect

$$-\frac{\partial x_1^*}{\partial E}\cdot\underbrace{\frac{\partial E}{\partial P_1}}_{\text{Shepard's Lemma}} = -\frac{\partial x_1^*}{\partial I}\cdot x_1^c = -\frac{\partial x_1^*}{\partial I}x_1^*$$

If normal good: $-\frac{\partial x_1^*}{\partial I} < 0$

inferior good : $-\frac{\partial x_1^*}{\partial I} > 0$

Lecture 8

Uncompensated Marshallian Demand

Maximize utility subject to budget constraints $\quad x_1^*(P_1, P_2, I)$

As $P_1 \downarrow$ we shift to higher indifference curves

Income + Substitution effects are present

Compensated Hicksian Demand

Minimize expenditure subject to utility constraint $\quad x_1^c(P_1, P_2, \bar{u})$

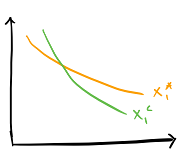
As $P_1\downarrow$ $I\downarrow$ while maintaining utility

Only substitution effects

$$x_1^c(P_1, P_2, \bar{u}) = x_1^*[P_1, P_2, E(P_1, P_2, \bar{u})]$$

Demand to reach minimum utility is equivalent to demand maximizing utility s.t. expenditure for Hicksian demand

Compensated demand is less responsive to price changes than uncompensated demand since it lacks income effects



Slutsky's Equation

$$\frac{\partial x_1^c}{\partial p_1} = \frac{\partial x_1^*}{\partial p_1} + \frac{\partial x_1^*}{\partial E} \cdot \frac{\partial E}{\partial p_1} \quad \leftarrow \text{differentiating earlier equality}$$

$$\frac{\partial x_1^*}{\partial p_1} = \underbrace{\frac{\partial x_1^c}{\partial p_1}}_{\text{Substitution effect}} - \underbrace{\frac{\partial x_1^*}{\partial E} \cdot \frac{\partial E}{\partial p_1}}_{\text{Income effect}}$$

Income effect $= -\frac{\partial x_1^*}{\partial I} x_1^*$

− for normal good

+ for inferior good

Labor Supply

Labor: exchange of time and effort for wages

Everything not considered labor is considered leisure (h)

Utility is a function of consumption and leisure

$$U = f(c, h)$$

Indifference curves are downward sloping

IC get flatter as h increases

Steeper ICs indicate a stronger preference for leisure

Worker's Opportunity Set (Budget constraint)

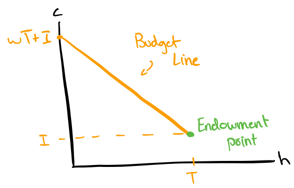
$$c = w(T-h) + I$$

w : hourly wage rate

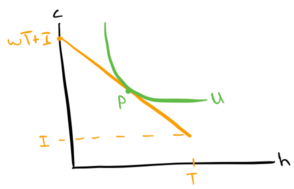
T: total hours

h: leisure hours

I : non labor Income



Workers choose c, h to maximize utility



at P, $MRS = \dfrac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial c}} = w$

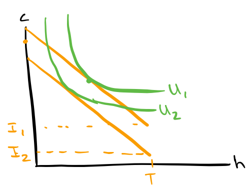
On the IC $dU = 0$

$$dU = \frac{\partial U}{\partial c} dc + \frac{\partial U}{\partial h} dh = 0$$

$$\frac{dc}{dh} = -\frac{\partial U / \partial h}{\partial U / \partial c} = -w$$

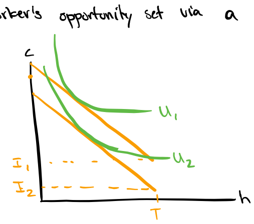
$$c = w(T-h) + I$$

An increase in non-labor income expands a worker's opportunity set via a parallel shift upwards



If leisure is normal

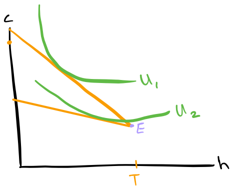
h↑



If leisure is inferior

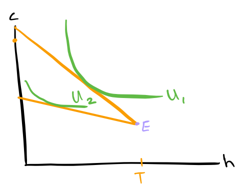
h↓

An increase in wage rate expands the worker's opportunity set and rotates the budget line around the endowment point



If leisure is normal

h↓



If leisure is normal

h↑

Lecture 9

When w increases (h is a normal good),

Income effect is positive

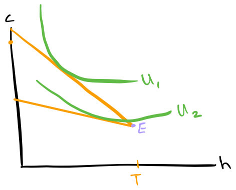
Worker wants to enjoy rewards of income

increases demand of normal goods

Substitution effect is negative

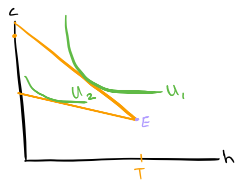
Leisure becomes more expensive

Reduced demand for leisure



Substitution > Income

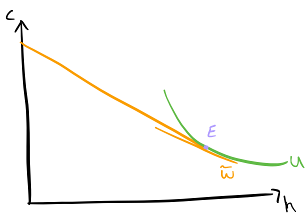
$\frac{\partial \ell^*}{\partial w} > 0$



Income > Substitution

$\frac{\partial \ell^*}{\partial w} < 0$

Consider the wage at which optimum utility is at the point of endowment



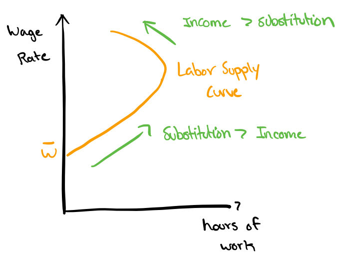
optimal work hours vs. wage

At wages higher than $\bar{w}$ we say an individual enters the labor market

It is worthwhile to work

Labor supply curve is positively sloped when substitution effect dominates income effect

optimal work hours vs. wage



Market Labor Supply curve is constructed by adding the supply curves of each worker

Reservation wage ($\bar{w}$) is the lowest wage rate that would make the individual indifferent to working

Find via MRS at the endowment point

Slutsky's Equation

$$h^c(w,1,\bar{u}) = h^*(w,1,E(w,1,\bar{u}))$$

$$\frac{\partial h^*}{\partial w} = \frac{\partial h^c}{\partial w} + (T-h^*)\frac{\partial h^*}{\partial I}$$

Lecture 10

Intertemporal Decisions: Money today is valued differently than money tomorrow

Discount rates or the "price of time"

Demand and supply in the market for future goods establishes a rate of return $r > 0$

Ex. \$50,000/year for 20 years

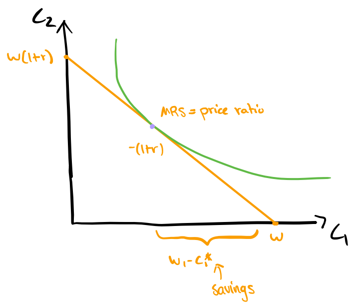
Worth today: $50{,}000 \sum_{t=0}^{20} \left(\frac{1}{1+r}\right)^t$

We can model intertemporal consumption via $U(c_1, c_2)$ where c_1 and c_2 are consumption bundles at differing timesteps

Still subject to a budget constraint

$$\max_{c_1, c_2} U(c_1, c_2) \quad \text{s.t.} \quad c_1 + \frac{c_2}{1+r} = W$$

$c_2 = (W - c_1)(1+r)$



Wealth not consumed is invested at rate r

price of future consumption is $1/1+r$

$$MRS = \frac{\text{current}}{\text{future}} = \frac{1}{1/1+r} = 1+r$$

Suppose wealth changes with time (I_1 in period 1 and I_2 in period 2)

Budget Constraint: $c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$

all future values must be discounted

Slutsky's Equation for intertemporal consumption

$$\frac{\partial c_1^*}{\partial r} = \frac{\partial c_1^*}{\partial r} + \left[\frac{c_2^*}{(1+r)^2}\right] \frac{\partial c_1^*}{\partial I}$$

Substitution effect is negative

pushes consumption down and savings up

Future consumption is cheaper

Income effect is positive (assuming consumption is normal)

Increase in r values todays income worth more

Drives consumption up and savings down

Lecture 11

Exam Review

Lecture 12

Attitudes towards Risk

- Risk Averse: unwilling to make a fair bet

  u is concave, $u'' \leq 0$

  decreasing marginal utility for money

- Risk Neutral: Indifferent about making a fair bet

  u is linear, $u'' = 0$

- Risk Seeking: Willing to make a fair bet

  u is convex, $u'' \geq 0$

Income vs. Excise Tax

Income tax is a reduction in the consumers income

$$I \to I(1-t)$$

t: income tax rate

Income tax shifts budget line but doesn't change marginal rate of substitution

Excise tax is an increase in price of the given good

$$P_2 \to (1+r)P_2$$

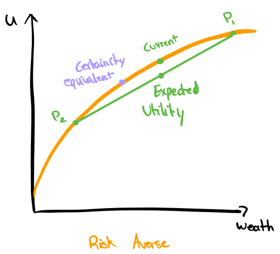
r: excise tax rate

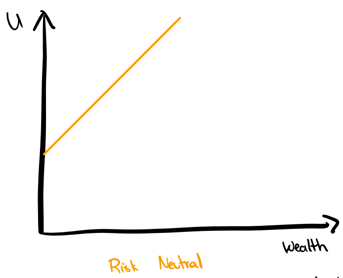
Excise tax changes both the optimal consumption bundle and the marginal rate of substitution

Excise taxes evoke behavioral changes

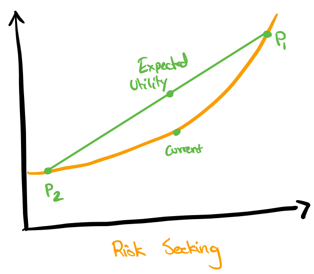
Lecture 13

Risk Attitudes





*Selects option with highest expected utility



Certainty equivalent is the guaranteed amount of money an individual would view as equally desirable as a risky asset

Risk premium is the difference between expected value and certainty equivalent

Arrow-Pratt Measure of Risk Aversion

$$-\frac{u''(x)}{u'(x)} \leftarrow \text{positive for risk adverse individuals}$$

For a risk decision

$$U(x_1, x_2) = \pi_1 u(x_1) + \pi_2 u(x_2)$$

$$MRS = \frac{\pi_1 u'(x_1)}{\pi_2 u'(x_2)}$$

Lecture 14

Consider the toy example

State	Probability	Consumption
1	π_1	$x_1 = w$
2	π_2	$x_2 = w - L$

Without insurance expected utility is simply

$$U = \pi_1 u(w) + \pi_2 u(w-L)$$

Assume $u' > 0$ and $u'' < 0$ (risk averse)

Insurance Policy Model

Premium Pz and claim Cz ← z units of insurance

State	Probability	Consumption
1	π_1	$x_1 = w - Pz$
2	π_2	$x_2 = w - L - Pz + Cz$

Expected Utility

$$U = \pi_1 u(w - Pz) + \pi_2 u(w - L - Pz + Cz)$$

Assume the policy is actuarially fair

$$\pi_1(Pz) + \pi_2(Pz - Cz) = 0$$

$$P = \pi_2 C$$

Approach 1: Consumer demand for insurance

$$\max_z \ \pi_1 u(\underbrace{w - Pz}_{x_1}) + \pi_2 u(\underbrace{w - L - Pz + Cz}_{x_2})$$

F.O.C.

$$\pi_1 u'(x_1)(-P) + \pi_2 u'(x_2)(C-P) = 0$$

$P = \pi_2 C$

$$\pi_2 u'(1-\pi_2) C = \pi_1 u'(x_1)\pi_2 C$$

$$u'(x_1) = u'(x_2)$$

Since u' is strictly increasing

$$x_1 = x_2$$

$$w - \pi_2 Cz = w - L - \pi_2 Cz + Cz$$

$$L = Cz \leftarrow \text{Full Insurance}$$

Approach 2: Solve each expression for z

$$x_1 = w - Pz \iff z = \frac{w - x_1}{P}$$

$$x_2 = w - L - Pz + Cz \iff z = \frac{w - L - x_2}{P - C}$$

$$\frac{w - x_1}{P} = \frac{w - L - x_2}{P - C}$$

$P = \pi_2 C$ and $\pi_1 + \pi_2 = 1$

$$\frac{w - x_1}{\pi_2 C} = \frac{w - L - x_2}{(\pi_2 - 1)C} \iff \pi_1(w - x_1) + \pi_2(w - L - x_2) = 0$$

Optimization

$$\max_{x_1 x_2} \ U(x_1, x_2) \quad \text{s.t.} \quad \pi_1(w - x_1) + \pi_2(w - L - x_2) = 0$$

Lecture 15

Risk pooling requires risks to be independent

General equilibrium is the study of consumer and supplier dynamics in whole economies

Economic efficiency is an economic state in which every resource is allocated to each person in the best way possible

General Equilibrium Model of Competitive Exchange Economy

- Consumers A, B, ... indexed by θ
- Goods 1, 2, ... n indexed by i
- Endowments denoted by w_i^θ

Determine

- prices $P_1 \cdots P_n$
- Allocation x_i^θ

Conditions for a perfectly competitive market

- Large number of Homogeneous goods
- Each good has an equilibrium price
- No transaction or transportation costs
- All parties have perfect information

Individuals are price takers

Each individual maximizes their utility by picking the bundle tangent to budget line and on IC

- MRS are equal among consumers
- prices adjust until quantity supplied equals demand

Budget constraints: Each consumer cannot spend more than they earn

Resource constraints: Economy can't consume more than it has

Lecture 16

In competitive equilibrium the ICs of both types of consumers are tangent at the same bundle on the price line

Welfare Theorem: Any competitive equilibrium is Pareto efficient

All mutually beneficial trades will occur

Competitive equilibrium depends on the endowment

initial distribution of wealth

Contract Curve is the set of all Pareto-efficient bundles

Any bundle where consumer ICs are tangent lie on the contract curve

Position on pareto curve is determined by bargaining abilities

Welfare Theorem: Any pareto efficient equilibrium can be obtained by competition for a sufficiently manipulated endowment

Lecture 17

Production economies

- A collection of goods and a collection of price-taking consumers
- A collection of price-taking firms
  produce goods using goods as inputs
- Each consumer is endowed with an initial bundle of goods and owns a share of each firm

Competitive Equilibrium for a production economy

- prices of all goods
- Consumers demand for each good
- Firms supply of each good

Subject to

- Consumers maximize utility
- Firms maximize profits
- Market clearing: supply equals demand

Consumers can now use their endowments as inputs for production

Since consumers are share holders, profits enter budget constraints

Production function shows the maximum amount of a good that can be produced using combinations of the inputs

Example: 2 consumers, 2 goods, and 2 Firms

Consumer A, Consumer B $\Big\}$ $U(x_1,x_2) = x_1^{1/2} x_2^{1/2}$

Assumptions: Each consumer is endowed with 14 units of time

Also owns half of each firm

Firm 1 $\downarrow$ x_1 Firm 2 $\downarrow$ x_2

$Y_1 = 2\sqrt{\ell_1}$ ← labor $Y_2 = 2\sqrt{\ell_2}$

price: P_1 price: P_2

output

a) Firm 1: $\max_{\ell_1} P_1 Y_1 - w\ell_1 = 2P_1\sqrt{\ell_1} - w\ell_1$

F.O.C: $P_1 \ell_1^{-1/2} = w$

Let $w = 1$

$\ell_1^* = P_1^2 \implies Y_1^* = 2P_1$

profit → $\pi_1^* = 2P_1^2 - P_1^2 = P_1^2$

Similarly, $\pi_2^* = P_2^2$

Consumer A's total income: $w \cdot 14 + \frac{\pi_1}{2} + \frac{\pi_2}{2} \overset{w=1}{=} 14 + \frac{P_1^2}{2} + \frac{P_2^2}{2}$

$$\max\ U(x_1^A, x_2^A) = x_1^{A\,1/2} x_2^{A\,1/2} \quad \text{s.t.} \quad P_1 x_1^A + P_2 x_2^A = \underbrace{14 + \frac{P_1^2}{2} + \frac{P_2^2}{2}}_{W_A}$$

Cobb Douglas ↳ $x_1^{A*} = \frac{W_A}{2P_1}$, $x_2^{A*} = \frac{W_A}{2P_2}$

$x_1^{B*} = \frac{W_B}{2P_1}$, $x_2^{B*} = \frac{W_B}{2P_2}$

Markets:

Good 1: $x_1^A + x_1^B = Y_1$ ⟶ $\frac{W_A}{2P_1} + \frac{W_B}{2P_1} = 2P_1$

Good 2: $x_2^A + x_2^B = Y_2$

Labor: $l_1 + l_2 = 14 + 14$

$$\text{Good 1}: \quad \frac{14 + \frac{P_1^2}{2} + \frac{P_2^2}{2}}{2P_1} + \frac{14 + \frac{P_1^2}{2} + \frac{P_2^2}{2}}{2P_1} = 2P_1$$

$$14 + \frac{P_1^2}{2} + \frac{P_2^2}{2} = 2P_1^2$$

$$\text{Good 2}: \quad 14 + \frac{P_1^2}{2} + \frac{P_2^2}{2} = 2P_2^2$$

$\therefore$ $P_1 = P_2$

$P_1^2 = 14$

$P_1 = \sqrt{14} = P_2$

Comparative advantage is the ability to produce a good at a lower cost than someone else

Production Possibility Fronteir: max combinations of two goods given a certain input

Slope is the marginal rate of transformation

Curve becomes smoother as more individuals are included

A single person would pick a product mix along the PPF that maximized utility

highest IC tangent to price line and PPF

Generalizes to multiple people

Lecture 18

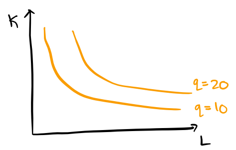
Production functions show the maximum amount of a good that can be produced

$q = f(x_1, \dots x_n)$

Marginal product of x_i where $i = 1, 2 \dots n$

$MP_{x_i} = \frac{\partial f}{\partial x_i}$ ← additional output that can be produced

Isoquant curve is the combinations of goods (L, K) that can produce a given q



Slope gives marginal rate of technical substitution

rate you can substitute K for L to keep q constant

$$dq = 0 = \frac{\partial f}{\partial L} dL + \frac{\partial f}{\partial K} dK$$

$$\frac{dK}{dL} = \underbrace{-\frac{MP_L}{MP_K}}_{MRTS}$$

Types of Costs

- Fixed cost is independent of output level
  loan payments, rent, etc.
- Variable cost
  inputs / related to wage bill

Total Cost = Fixed Cost + Variable Cost

- Average Variable Cost

$$AC = \frac{VC(w,r,q)}{q}$$ cost per unit output

- Marginal Cost

$$MC = \frac{\partial VC(w,r,q)}{\partial q}$$ Cost of one additional unit

Profit

$$\pi = \underbrace{R(q)}_{\text{Revenues}} - \underbrace{(P_1 X_1 + \dots P_n X_n)}_{\text{Costs of all inputs}}$$

Consumers maximize individual utilities $\longrightarrow$ Aggregate Individual behavior determines market demand

Firms maximize profits

Assuming no fixed costs

$$\pi = p(q)q - wL - rk$$

Labor, capital

Perfectly Competitive

$$\pi = pq - wL - rk$$

Profit maximization: input-oriented

$$\max_{L,K} \pi = R(f(L,K)) - wL - rK$$

wage, price to capital

$$\frac{\partial \pi}{\partial L} = 0 \longrightarrow \frac{\partial R}{\partial q}\frac{\partial f}{\partial L} - w = 0 \Rightarrow \frac{\partial R}{\partial q} = \frac{w}{MP_L}$$

$$\frac{\partial R}{\partial q} MP_L - w = 0$$

$$\frac{\partial \pi}{\partial K} = 0 \longrightarrow \frac{\partial R}{\partial q} \cdot \frac{\partial f}{\partial K} - r = 0 \Rightarrow \frac{\partial R}{\partial q} = \frac{r}{MP_K}$$

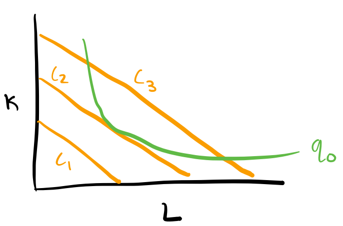
$$\frac{\partial R}{\partial q} MP_K - r = 0$$

marginal revenue product = price

$$\frac{w}{MP_L} = \frac{r}{MP_K} \Leftrightarrow \frac{MP_L}{MP_K} = MRTS = \frac{w}{r}$$

Cost Minimization: Output-oriented approach

$$\min_{L,K} wL + rK \quad \text{s.t.} \quad f(L,K) = \bar{q}$$



$$\mathcal{L}(L,K,\lambda) = wL + rK + \lambda[f(L,K) - \bar{q}]$$

First Order Conditions

$$\frac{\partial \mathcal{L}}{\partial L} = 0 \longrightarrow w - \lambda MP_L = 0 \Rightarrow \lambda = \frac{-w}{MP_L}$$

$$\frac{\partial \mathcal{L}}{\partial K} = 0 \longrightarrow r - \lambda MP_K = 0 \Rightarrow \lambda = -\, r/MP_K$$

$$\frac{MP_L}{MP_K} = MRTS = \frac{w}{r}$$

Marginal productivity per dollar spent is equal for all inputs

$$\frac{MP_L}{w} = \frac{MP_K}{}$$

Negative of the Lagrange multiplier equals marginal cost

$$\frac{\partial \mathcal{L}}{\partial \bar{q}} = -\lambda$$

Returns to Scale

$$f(tL, tK) = t^k f(L, K)$$

If $k > 1$: Increasing returns to scale

output increases more than proportionately

Economies of Scale: decreasing average costs

If $k < 1$: Decreasing returns to scale

output increases less than proportionately

Disecono mies of Scale: increasing average costs

If $k = 1$: Constant Returns to Scale

Output increases proportionately

Linear production functions

Marginal cost is equal to average cost at minimum point

Lecture 19

Firm's supply curve is given by the appropriate part of its MC curve

$$P > AC(q_i)$$

Market supply is given by the sum of the firms' supply curves

Time Horizon's determine outcomes in a competitive market

Short Run

- Some inputs are fixed + some are variable
- Firms are unable to enter or exit the market
- Profits may be positive or Negative

Long Run

- All inputs are variable
- Firms can enter + exit
- In a competitive market firms earn 0 profits

ensures efficiency in production

Economic Monopolies

Supported by barriers to entry

Natural Monopoly: One firm can produce total output of market at lower cost than several firms

capital costs predominate so economies of scale are created

Legal Monopoly: Patents

Input Monopoly: Only supplier of a good where there is no close substitute

Monopolies can set its price

$$\pi(Q) = TR(Q) - C(Q) = p(Q)Q - C(Q)$$

First Order Conditions

$$\frac{d\pi}{dQ} = 0 \implies MR = MC$$

$$\frac{dp}{dQ} Q + p = MC$$

$$p\left(\frac{dP}{dQ} \cdot \frac{Q}{P} + 1\right) = MC$$

$$p\left(-\frac{1}{\varepsilon_D} + 1\right) = MC$$

$$-\frac{dQ}{dp}\frac{P}{Q}$$

$\varepsilon_D > 1$ so the monopoly chooses an output on the elastic portion of the demand curve

Lerner Index is a measure of market power

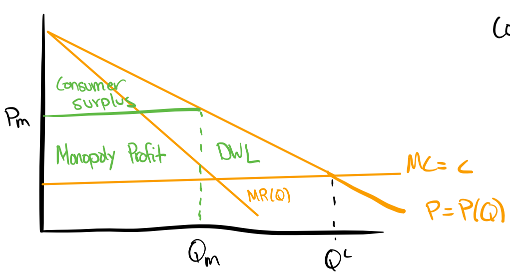
$$L = \frac{p - MC}{p}$$

Lecture 20

Efficiency of Perfect Competition

For a price of p and consumption of 1 unit

Everyone with willingness-to-pay higher than p will buy the good



Consumer Surplus is the difference between the total amount that consumers are WTP and the total amount they actually pay

Dead weight loss is the loss in consumer surplus due to users with

$P_m > WTP > MC$ (Excluded by monopoly pricing)

$$DWL = \frac{1}{2}(P_m - c)(Q_c - Q_m)$$

Producer Surplus is the benefit the producer recieves for selling the good

Difference between amount producer recieves and amount they were willing to accept for the good

Price Discrimination

Charging different prices for different units of the same good

No Arbitrage!

1-st Degree (Perfect) Price Discrimination

monopolist knows each consumers WTP

Charge each consumer their WTP

DWL = 0 but consumers are left with 0 surplus

3rd Degree Price Discrimination

monopolist observes consumers' characteristics + charges different prices based on characteristics

Differentiate price between groups

Two-Segment Example

profit : $\pi(q_1, q_2) = P_1(q_1)q_1 - cq_1 + P_2(q_2)q_2 - cq_2$

Costs / MC are constant

First Order Conditions

$$\frac{\partial \pi}{\partial q_1} = 0 = \frac{\partial \pi}{\partial q_2} \qquad MR_1(q_1) = c = MR_2(q_2)$$

Marginal revenues are equal across segments

$$MR(q) = p + \frac{dp}{dq}p = p\left(1 - \frac{1}{\varepsilon_D}\right)$$

$$P_1\left(1 - \frac{1}{\varepsilon_1}\right) = c = P_2\left(1 - \frac{1}{\varepsilon_2}\right)$$

If $\varepsilon_1 > \varepsilon_2$ then $P_1 < P_2$ so the monopolist charges higher prices in less elastic markets

less responsive to higher prices

Versioning is a technique used when groups of buyers are hard to identify

offer different products and let buyers select

Oligopoly

More than 1 firm but few enough firms that each recognizes its impact on price

Imperfect competition usually involves strategic interactions

Nash Equilibrium : Each firm chooses the best strategy given strategies chosen by the other firm

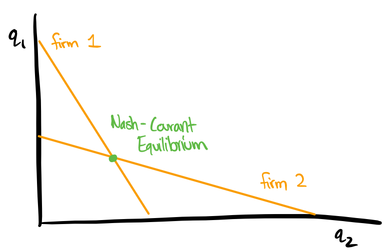
Lecture 21

Nash Equilibrium: A profile of strategies $s_i, \dots s_\ell$ s.t. for each player i, s_i is the best response to the strategies of the other players

No player has incentive to deviate from this strategy profile

production

Consider the case where competitors simultaneously pick quantities



Best-response curves slope downwards since as the rival increases quantity, the firm decreases theirs

$P^* > MC$: market power

Each firm acts as a monopolist on its own residual demand curve

demand not met by other sellers

Collusive Agreement: Agreement between 2+ companies to restrict output, raise the price, and increase profits

Two-firms can act as a monopoly and split their profits

Can calculate collusion vs. competition profits

Incentives to cheat exist when one firm would profit by producing more

Assume competitor produces collusion amounts

Write pay-offs in matrix

		Firm 2	
		Collude	Defect
Firm 1	Collude	π_1, π_2	π_1, π_2
	Defect	π_1, π_2	π_1, π_2

Lecture 22

Suppose firms interact repeatedly

Trigger Strategies

1. Firm 1 colludes as long as Firm 2 does
2. If firm 1 cheats, then firm 2 pulls the "trigger" to punish firm 1

Cheating produces short-run profits and long-term punishment

Payoff for adhering

$$P_{collusion}\left(1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots\right) = P_{collusion}\frac{(1+r)}{r}$$

Payoff for cheating

both cheat

$$P_{cheat} + \frac{P_{punish}}{1+r}\left(1 + \frac{1}{1+r} + \cdots\right) = P_{cheat} + \frac{P_{punish}}{r}$$

Firms only honor the agreement if

$$P_{collusion}\frac{(1+r)}{r} > P_{cheat} + \frac{P_{punish}}{r}$$

Cournot Competition: n firms

- $n > 2$
- homogeneous goods
- Simultaneously pick strategies
- $C_i(q_i)$ is the same for each i
- Demand $p(Q)$ is a function of $Q = \sum_{i=1}^{n} q_i$

Firm 1 maximizes

$$\pi_1 = p(Q)\, q_1 - C_1(q_1)$$

FOC:

all but q_1

$$\frac{\partial \pi_1}{\partial q_1} = 0 \longrightarrow \text{function relating } q_1 \text{ to } Q_{-1}$$

Impose symmetry: $q_i^* = q^*$ so $Q_{-1}^* = (n-1)q^*$

Each firm has the same reaction function

Cournot solution reproduces results from monopoly, duopoly and perfect competition

Lecture 23

Suppose there is a market with 2 firms

1. Firm 1 moves first and selects q_1
2. Firm 2 moves next and selects q_2 in response to q_1

Solve the problem by solving q_2 in terms of q_1

Assume firm 1 selects q_1 with the knowledge of $q_2 = f(q_1)$

Leader will be better off

Externalities

Relaxing the assumption that actions of a firm or individual do not impact others in the economy

Costs are negative externalities

Benefits are positive externalities

Consider 2 firms with profits

$$\pi_1 = P_1 q_1 - C_1(q_1, z)$$

$$\pi_2 = P_2 q_2 - C_2(q_2, z)$$

z is an activity or input chosen by firm 1 that adversly impacts firm 2

z reduces firm 1's costs and increases firm 2's

$$\frac{\partial C_1}{\partial z} < 0 \quad \text{and} \quad \frac{\partial C_2}{\partial z} > 0$$

Firm 1 chooses the pollution level disregarding external effect on firm 2

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} > 0 \quad \leftarrow \text{inefficient outcome}$$

Approach 1: Regulation

Impose limits so that

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

directly : Limits

Indirectly : Requirements for pollution reduction equipment

constraint : $z \leq \bar{z}$

$$\frac{\partial C_1(q_1, \bar{z})}{\partial z} + \frac{\partial C_2(q_2, \bar{z})}{\partial z} = 0$$

Approach 2: A market for clean air

- Firm 2 owns the right to clean air
- Firm 1 must pay for the right to pollute

New profit equations

$$\pi_1 = P_1 q_1 - P_z z - C_1(q_1, z)$$

$$\pi_2 = P_2 q_2 + P_z z - C_2(q, z)$$

FOC w.r.t. z

$$\frac{\partial \pi_1}{\partial z} = 0 \longrightarrow -P_z = \frac{\partial C_1}{\partial z}$$

$$\frac{\partial \pi_2}{\partial z} = 0 \longrightarrow P_z = \frac{\partial C_2}{\partial z}$$

$$\longrightarrow \frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

Efficient outcome!

Approach 3: Market for pollution

Firm 1 has the right to pollute up to some maximum $\bar{z}$

New profit equations

$$\pi_1 = P_1 q_1 + P_z(\bar{z} - z) - C_1(q_1, z)$$

$$\pi_2 = P_2 q_2 - P_z(\bar{z} - z) - C_2(q_2, z)$$

FOC w.r.t. z

$$\frac{\partial \pi_1}{\partial z} = 0 \longrightarrow -P_z = \frac{\partial C_1}{\partial z}$$

$$\frac{\partial \pi_2}{\partial z} = 0 \longrightarrow P_z = \frac{\partial C_2}{\partial z}$$

$$\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z} = 0$$

Approach 4: Merger

If the firms merge, they "internalize" the externality

$$\pi = P_1 q_1 - C_1(q_1, z) + P_2 q_2 - C_2(q_2, z)$$

FOC w.r.t z

$$\frac{\partial \pi}{\partial z} = 0 \longrightarrow -\frac{\partial C_1}{\partial z} - \frac{\partial C_2}{\partial z} = 0$$

Approach 5: Taxation

Tax the externality

$$\pi_1 = P_1 q_1 - C_1(q_1, z) - tz$$

FOC w.r.t z

$$\frac{\partial \pi_1}{\partial z} = 0 \longrightarrow -\frac{\partial C_1}{\partial z} = t$$

Outcome is efficient if

$$\pi_2 = P_2 q_2 - C_2(q_2, z) + tz$$

where

$$\frac{\partial \pi_2}{\partial z} = 0 \longrightarrow \frac{\partial C_2}{\partial z} = t$$

Lecture 24

How does asymmetric information affect individual/market behaviour?

Hidden Characteristics: An attribute that is known only to a subset of the population

Hidden Action: An act by one party to a transaction that is unobserved by others

Many economic transactions can be understood in the principal-agent framework

- Agent is responsible for carrying out a task whose success is dependent on effort
- Principal benefits from effort used on this task but cannot observe effort

Solution: Principal offers wage-contract that incentivizes agent to exert effort at the lowest possible cost

Price rates pay worker according to a measure of output

Align worker and owner objectives through

- Commissions
- Profit-sharing
- Bonuses

Tournaments reward based on relative ranking not absolute output

can encourage too much competition

Incentivizes sabatoge