Intermediate Microeconomics Final Review

Unconstrained Optimization

$$\max_{x_1,x_2} U(x_1,x_2) \longrightarrow \frac{\partial u}{\partial x_1} = \frac{\partial u}{\partial x_2} = 0$$

$$H = \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial x \partial y} \\ \frac{\partial u}{\partial y \partial x} & \frac{\partial^2 u}{\partial y^2} \end{bmatrix}$$

Maximum: $\frac{\partial^2 u}{\partial x_1^2} < 0$ and $|H| > 0$

Minimum: $\frac{\partial^2 u}{\partial x_1^2} > 0$ and $|H| > 0$

Homogeneity

$$f(tx_1, tx_2, \dots tx_n) = t^k f(x_1, x_2, \dots x_n)$$

Homogeneous of degree k

Indifference Curves

Upper contour set is the set of consumption bundles with utility greater than U^*

Monotonic preferences always increase as quantity increases

Quasiconcavity means the average of two bundles on an IC exists in the upper contour set

roughly consumer preference for diverse consumption bundles

Prove quasiconcavity by proving each IC is convex

$$f'' > 0$$

You can ignore second order conditions when U is quasiconcave and constraint is linear

Marginal Rate of Substitution is the negative slope of the IC

diminishing MRS is when higher x_1 results in willingness to sacrifice x_1 for x_2

$$MRS = \frac{-\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{\text{Marginal Utility 1}}{\text{Marginal Utility 2}}$$

$$MRS|_{optimum} = P_1/P_2$$ ← Individual Consumer matches market pricing

Homothetic Utility Functions

Utility functions that are increasing transformations of eachother yield the same demand

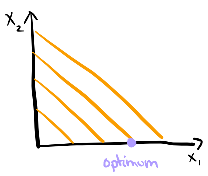
V is said to be an increasing transformation of U if $V = f(U)$ and $f' > 0$

Special Goods

Perfect Substitutes : Constant rate of substitution

$$U(x_1,x_2) = ax_1 + bx_2 \quad \text{for } a,b > 0$$

$$MRS = a/b$$

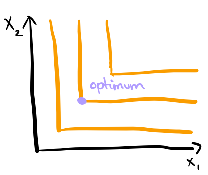


Perfect Substitutes yield corner solutions

Perfect Complements : Consumer consumes goods in constant proportions

$$U(x_1,x_2) = \min\{ax_1, bx_2\} \quad \text{for } a,b > 0$$

$$MRS = 0 \text{ or infinity}$$



Constrained Optimization (Lagrange Multipliers)

$$\max_{x_1,x_2} U(x_1,x_2) \quad \text{s.t.} \quad P_1x_1 + P_2x_2 = I$$

$$L = U(x_1,x_2) + \lambda(P_1x_1 + P_2x_2 - I)$$

First Order Conditions

$$\frac{\partial L}{\partial x_1} = 0 \quad \frac{\partial L}{\partial x_2} = 0 \quad \frac{\partial L}{\partial \lambda} = 0$$

1) Solve for λ, set expressions equal
2) plug in expression into constraint eq

$$\lambda = \frac{-\frac{\partial u}{\partial x_1}}{P_1} = \frac{-\frac{\partial u}{\partial x_2}}{P_2} = \frac{\text{Marginal Benefit}}{\text{Marginal Cost}}$$ ← Shadow Value

At optimum, all goods have the same $\frac{\text{Marginal Benefit}}{\text{marginal cost}}$

Second Order Conditions

$$B = \begin{vmatrix} \frac{\partial^2 L}{\partial \lambda^2} & \frac{\partial^2 L}{\partial \lambda \partial x_1} & \frac{\partial^2 L}{\partial \lambda \partial x_2} \\ \frac{\partial^2 L}{\partial x_1 \partial \lambda} & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} \\ \frac{\partial^2 L}{\partial x_2 \partial \lambda} & \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} \end{vmatrix}$$

Maximum: $\begin{vmatrix} L_{\lambda\lambda} & L_{\lambda\lambda} \\ L_{1\lambda} & L_{11} \end{vmatrix} < 0$

$|B| > 0$

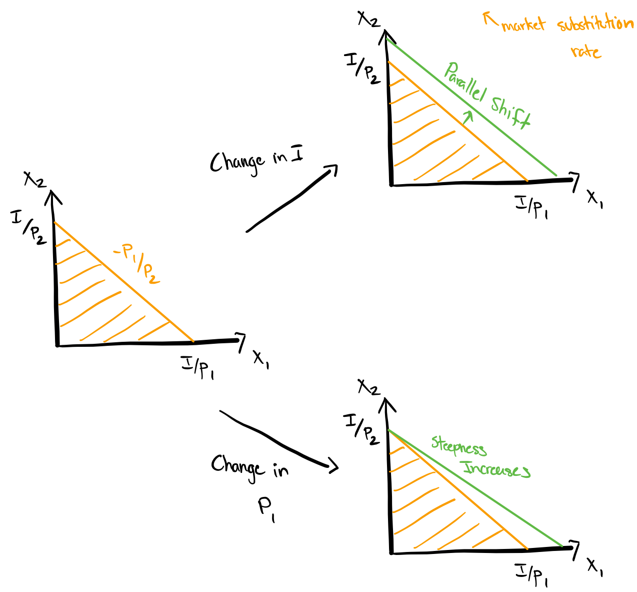
Minimum: $\begin{vmatrix} L_{\lambda\lambda} & L_{\lambda 1} \\ L_{1\lambda} & L_{11} \end{vmatrix} < 0$

$|B| < 0$

Budget Set

Consider $I = P_1x_1 + P_2x_2 \longrightarrow x_2 = -\frac{P_1}{P_2}x_1 + I/P_2$

↖ market substitution rate

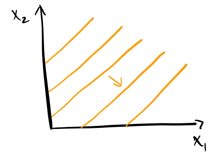


Elasticity

How does P_1 effect total expenditure on x_1?

$$\frac{dP_1 x_1^*(P_1)}{dP_1} = x_1^*(1 - \varepsilon_P) \qquad \varepsilon_P = -\frac{dx_1^*}{dP_1} \cdot \frac{P_1}{x_1^*}$$

Bads: Indifference Curves with negative slopes



Substitutes vs. Complements

x_2 is a substitute for x_1 if $\frac{dx_2^*}{dp_1} > 0$

x_2 is a complement for x_1 if $\frac{dx_2^*}{dp_1} < 0$

Expenditure Minimization

$$\min \; p_1x_1 + p_2x_2 \quad \text{s.t.} \quad U(x_1,x_2) = \bar{U}$$

$$L = p_1x_1 + p_2x_2 + \lambda(\bar{U} - U(x_1,x_2))$$

First Order Conditions

$$\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \frac{\partial L}{\partial \lambda} = 0$$

Shepard's Lemma

Compensated demand can be found from differentiating w.r.t price

$$\frac{\partial E(p_1,p_2,\bar{U})}{\partial p_1} = x_1^c(p_1,p_2,\bar{U})$$

Types of Demand

Uncompensated Marshallian Demand: Max utility given a budget constraint

Income and substitution effect captured

Compensated Hicksian Demand: Min expenditure for a given utility

Only captures substitution effect

Less responsive to changes

Slutsky's Equation

$$x_1^c(p_1,p_2,\bar{U}) = x_1^*[p_1, p_2, E(p_1,p_2,\bar{U})]$$

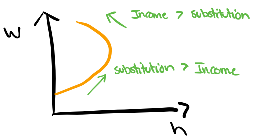
$$\frac{\partial x_1^c}{\partial p_1} = \frac{\partial x_1^*}{\partial p_1} + \frac{\partial x_1^*}{\partial E} \cdot \frac{\partial E}{\partial p_1}$$

$$\frac{\partial x_1^*}{\partial p_1} = \frac{\partial x_1^c}{\partial p_1} - \frac{\partial x_1^*}{\partial E} \cdot \frac{\partial E}{\partial p_1}$$

Shepard's Lemma

$$\frac{\partial x_1^*}{\partial p_1} = \underbrace{\frac{\partial x_1^c}{\partial p_1}}_{\text{Substitution effect}} - \underbrace{\frac{\partial x_1^*}{\partial I} \cdot x_1^*}_{\text{Income Effect}}$$

Labor Supply Curve



Intertemporal Consumption

$$\max_{c_1,c_2} U(c_1,c_2) \quad \text{s.t.} \quad c_1 + \frac{c_2}{1+r} = W \qquad MRS = \frac{\text{Current}}{\text{Future}} = 1+r$$

discount rate

Wealth not invested is invested at rate r

If wealth changes with time $\rightarrow$ $c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$

Unit Elastic ($\varepsilon_p = 1$)

$$\frac{dp_1x_1^*(p_1)}{dp_1} = 0$$

Change in price is equal and opposite to demand

No change in expenditure

Elastic ($\varepsilon_p > 1$)

$$\frac{dp_1x_1^*(p_1)}{dp_1} < 0$$

demand is sensitive to price

Decrease in expenditure

Inelastic ($\varepsilon_p < 1$)

$$\frac{dp_1x_1^*(p_1)}{dp_1} > 0$$

demand is resilliant to price

Increase in expenditure

Income Elasticity

$$\frac{dx_1^*}{dI} \; \frac{I}{x_1^*}$$

Goods and Effects

Substitution Effect: Stays on original IC

Income Effect: Shifts to a higher IC

Normal Good: Demand for x_1 increases as income increases

$$\frac{\partial x_1^*}{\partial I} > 0$$

Income and substitution effect reinforce eachother

$P_x \downarrow \; x^* \uparrow$

Inferior Good: Demand for x_1 decreases as income increases

$$\frac{\partial x_1^*}{\partial I} < 0$$

Income and substitution effects conflict

$P_x \downarrow \; x^* \downarrow$ $\qquad$ $P_x \downarrow \; x^* \uparrow$

Giffen Good: Demand for x_1 increases as price increases

Income effect outweighs substitution effects

Labor Supply

Define utility as a function of leisure and consumption

$$U = f(c,h)$$

Worker Opportunity Set ← Budget Constraint

$$c = w(T-h) + I$$

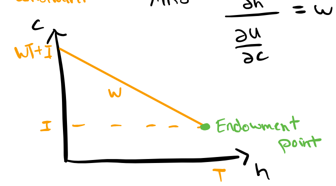
labor supply

w: hourly wage

T: total hours

h: leisure hours

I: non-labor Income



$$MRS = \frac{\frac{\partial U}{\partial h}}{\frac{\partial U}{\partial c}} = w$$

When w increases

Income effect is positive

worker wants to enjoy rewards of income

Substitution effect is negative

Leisure becomes more expensive

Reservation wage is the minimum wage at which it is worthwhile to work

Slutsky for Labor

$$\frac{\partial h^*}{\partial w} = \frac{\partial h^c}{\partial w} + (T-h^*)\frac{\partial h^*}{\partial I}$$

Slutsky's Equation for Intertemporal Consumption

$$\frac{\partial c_1^*}{\partial r} = \frac{\partial c_1^c}{\partial r} + \left[\frac{c_2^*}{(1+r)^2}\right]\frac{\partial c_1^*}{\partial I}$$

Substitution effect is negative

- Future consumption is cheaper
- pushes consumption down and savings up

Income effect is positive

increase in r values todays income more

Drives consumption up and savings down

Taxes

Income tax is a reduction of consumer income

$I \to I(1-t)$

Shifts budget line but doesn't change MRS

Excise Tax is an increase in the price of a good

$P_2 \to (1+r)P_2$

Shifts optimal consumption bundle and MRS

Evoke behavioral changes

Risk Attitudes

Risk Averse: unwilling to make a fair bet

u is concave, $u'' \le 0$

decreasing marginal utility for money

Risk Neutral: Indifferent about making a fair bet

u is linear, $u'' = 0$

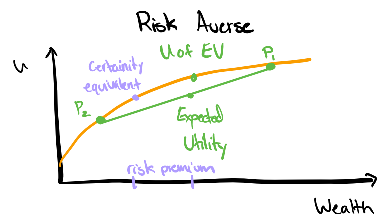
Risk Seeking: Willing to make a fair bet

u is convex; $u'' \ge 0$

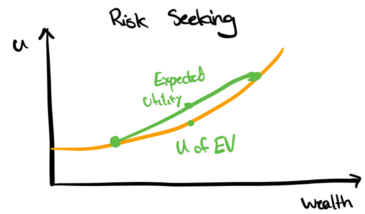
$$MRS = \frac{\pi_1 u'(x_1)}{\pi_2 u'(x_2)}$$

Arrow-Pratt

$$\frac{-u''(x)}{u'(x)}$$ ← positive for risk averse individuals



guaranteed amt of money equal to risky asset



Insurance

Insurance Policy Model

State	Probability	Consumption
1	π_1	$x_1 = W - Pz$
2	π_2	$x_2 = W - L - Pz + Cz$

Premium Pz and claim Cz

Expected Utility

$$U = \pi_1 U(W-Pz) + \pi_2 U(W-L-Pz+Cz)$$

Assuming actuarially fair

$$\pi_1(Pz) + \pi_2(Pz - Cz) = 0$$

$$P = \pi_2 C$$

Consumer Demand Approach

$$\max_z \ \pi_1 U(\underbrace{W-Pz}_{x_1}) + \pi_2 U(\underbrace{W-L-Pz+Cz}_{x_2})$$

F.O.C

$$\pi_1 u'(x_1)(-P) + \pi_2 u'(x_2)(C-P) = 0$$

$P = \pi_2 C$

$u'(x_1) = u'(x_2)$

$x_1 = x_2$

$L = Cz$ ← full insurance

Solving for Policy Count

$$x_1 = W - Pz \iff z = \frac{W - x_1}{P}$$

$$x_2 = W - L - Pz + Cz \to z = \frac{W-L-x_2}{P-C}$$

$$\frac{W-x_1}{P} = \frac{W-L-x_2}{P-C}$$

$P = \pi_2 C$

$$\pi_1(W-x_1) + \pi_2(W-L-x_2) = 0$$

$$\max_{x_1, x_2} U(x_1, x_2) \text{ s.t. } \pi_1(W-x_1) + \pi_2(W-L-x_2)$$

leads to full insurance solution

Exchange Economy

General Equilibrium model of Competitive Exchange Economy

- Consumers, goods and endowments determine prices and allocation
- Require large number of homogeneous goods, goods have equilibrium prices, no transaction/transportation costs, and all parties have perfect info

Individuals are price takers

- Each individual maximizes their utility by picking consumption bundle tangent to IC

Welfare Theorem: Any competitive equilibrium is pareto efficient

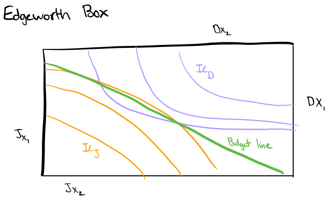
all mutually beneficial trades will occur

Contract curve is the set of all pareto-efficient bundles

Bundles where consumer IC's are tangent

Exchange Economy without Production

Edgeworth Box



Each consumer selects bundle tangent to budget line

$MRS_J = P_1/P_2 = MRS_D$

Budget Constraints:

$$P_1 x_1^D + P_2 x_2^D = P_1 w_1^D + P_2 w_2^D$$

$$P_1 x_1^J + P_2 x_2^J = P_1 w_1^J + P_2 w_2^J$$

Resource Constraint:

$$x_1^D + x_1^J = w_1^D + w_2^J$$

$$x_2^D + x_2^J = w_2^D + w_2^J$$

$\max U(x_1^D, x_2^D)$ s.t. $P_1 x_1^D + P_2 x_2^D = W_A$ for each participant

- Solve for demands by plugging λ expressions into constraint
- Solve for prices by setting one price to 1 and applying resource constraint

Economies w/ Production

- Collection of goods and a collection of price taking customers
- Collection of price taking firms that can produce some of these goods using other goods as inputs

· Each consumer is endowed with an initial amount of each goods and owns a share of each firm

Consumers maximize utility and firms maximize profits

Demand and Supply for all goods are equal (Market Clearing)

Example : 2 consumers, 2 goods, and 2 firms

Each consumer has 14 units of time and owns ½ of each firm

Consumer A, Consumer B $\Big\}$ $U(x_1, x_2) = x_1^{1/2} x_2^{1/2}$

Firm 1 produces x_1

Firm 2 produces x_2

Firm 1

output : $y_1 = 2\sqrt{\ell_1}$

price : P_1

$\max_{\ell_1} P_1 Y_1 - w\ell_1$

let $w = 1$

$\pi_1^* = P_1^2$

Firm 2

output : $Y_2 = 2\sqrt{\ell_2}$

price : P_2

$\max_{\ell_2} P_2 Y_2 - w\ell_2$

$\pi_2^* = P_2^2$

Consumer Income

$\underbrace{w \cdot 14 + \frac{\pi_1}{2} + \frac{\pi_2}{2}}_{W_A}$

Solve consumer maximization accordingly

Markets

Good 1 : $x_1^A + x_1^B = Y_1$

Good 2 : $x_2^A + x_2^B = Y_2$

Labor : $\ell_1 + \ell_2 = 14 + 14$

Solve each given earlier information

Production Possibility Frontier : max combinations of two goods given a certain input

Slope = marginal rate of transformation

Curve smoothens as participants increase

Isoquant curves are combinations of goods for a given production value

Slope gives marginal rate of technical substitution

Production and Costs

Average Variable Cost : $AC = \frac{VC}{q}$ cost per output

Marginal Cost : $MC = \frac{\partial VC}{\partial q}$ cost of one additional unit

Profit Maximization Approach (Input oriented)

$\max_{L,K} \ \pi = R(f(L,K)) - wL - rK$

$\frac{\partial \pi}{\partial L} = 0 \Rightarrow \frac{\partial R}{\partial q} = \frac{w}{MP_L}$ $\quad \frac{\partial \pi}{\partial K} = 0 \Rightarrow \frac{\partial R}{\partial q} = \frac{r}{MP_K}$

$MRTS = \frac{w}{r}$

Cost Minimization Approach (Output Oriented)

$\min_{L,K} \ wL + rK \quad \text{s.t.} \quad f(L,K) = \bar{q}$

F.O.C $\Rightarrow \lambda = -\frac{w}{MP_L} = -\frac{r}{MP_K}$ $\quad$ marginal cost $= -\lambda$

Returns to Scale

$f(tL, tK) = t^k f(L,K)$

If $k > 1$: Increasing economies of scale decreasing AC

if $k < 1$: Decreasing diseconomies of scale increasing AC

If $k = 1$: Constant Linear production functions

Perfect Competition

Each firm i : $\max_{q_i} \ pq_i - C(q_i) \Rightarrow p = MC(q_i^*)$

defines supply curve

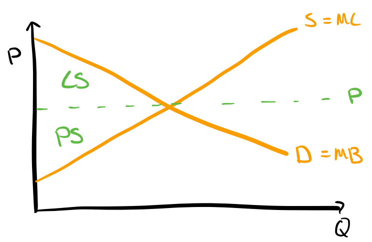
Market supply is the sum of each supply curve

Outcome in a market is given by firm supply

short run
- fixed and variable inputs
- Unable to enter/exit market
- Non-zero profits

long-run
- variable inputs
- firms can enter/exit
- Earn 0 profits

Efficiency



Deadweight loss is loss in consumer surplus from consumers unwilling to pay at pm

Monopolies

Natural Monopoly : One firm can produce market supply cheaper

Legal Monopoly : Patents

Input Monopoly : Only 1 supplier with no substitute

Monopolies set its price

$\pi(Q) = p(Q)Q - C(Q)$

F.O.C

$MR = MC$

$P(-1/\varepsilon_D + 1) = MC$ $\quad$ monopoly chooses an output on elastic portion of curve

$\varepsilon_D = -\frac{dQ}{dp} \cdot \frac{P}{Q}$

Lerner Index is a measure of market power

$L = \frac{P - MC}{P}$

Price Discrimination

Assumes no arbitrage

1st degree : Perfect Price Discrimination

Monopolist knows each consumers WTP

DWL = 0 but 0 consumer surplus

3rd Degree Price Discrimination

monopolist observes consumer characteristics

Two-Segment Example

profit : $\pi(q_1, q_2) = P_1(q_1) q_1 - cq_1 + P_2(q_2) q_2 - cq_2$

F.O.C. $\frac{\partial \pi}{\partial q_1} = 0 = \frac{\partial \pi}{\partial q_2} \Rightarrow MR_1(q_1) = c = MR_2(q_2)$

$P_1(1 - 1/\varepsilon_1) = c = P_2(1 - 1/\varepsilon_2)$

monopolist charges higher prices to less elastic markets

Oligopoly

More than 1 firm but enough that each impacts price

Nash equilibrium : each firm chooses the best strategy given strategies chosen by other firms

Collusion : Agreement between companies to collaborate

- Act as a monopoly and split profits

  calculate collusion vs. competition profits

- Cheat in an existing collusive agreement

  Assume competitor produces same amount

Organize Outcomes in payoff matrix

Trigger Strategies

Repeated interactions between firms

- Collude as long as they both do
- When 1 firm deviates, it is subsequently punished

Adhering Payoff $\quad \frac{P_{collusion}(1+r)}{r}$

Payoff for cheating $\quad P_{cheat} + \frac{P_{punish}}{r}$

Firms honor collusion agreement if

$P_{collusion} \frac{(1+r)}{r} > P_{cheat} + \frac{P_{punish}}{r}$

Cournot Competition

n firms

Firm 1 maximizes : $\pi_1 = p(Q) q_1 - C_1(q_1)$

F.O.C. $\frac{\partial \pi_1}{\partial q_1} = 0 \rightarrow$ function relating q_1 and Q_{-1}

Impose symmetry $q_1^* = q^*$ so $Q_{-1}^* = (n-1)q^*$

Same reaction function

Leader/Follower

1. Firm 1 moves first and selects q_1
2. Firm 2 moves next and selects q_2

Solve q_2 in terms of q_1

Assume firm 1 solves q_1 with knowledge $q_2 = f(q_1)$

Leader benefits!

Externalities

External interactions between firms

$\pi_1 = P_1 q_1 - C_1(q_1, z)$ $\quad \frac{\partial C_1}{\partial z} < 0$

$\pi_2 = P_2 q_2 - C_2(q_2, z)$ $\quad \frac{\partial C_2}{\partial z} > 0$

Approach 1: Regulation

Constraint : $z \le \bar{z}$

$\frac{\partial C_1(q_1, \bar{z})}{\partial z} + \frac{\partial C_2(q_2, \bar{z})}{\partial z} = 0$

Approach 2: A market for clean air

$\pi_1 = P_1 q_1 - P_z z - C_1(q_1, z)$

$\pi_2 = P_2 q_2 + P_z z - C_2(q_2, z)$

Approach 3: Market for pollution

$\pi_1 = P_1 q_1 + P_z(\bar{z} - z) - C_1(q_1, z)$

$\pi_2 = P_2 q_2 - P_z(\bar{z} - z) - C_2(q_2, z)$

Approach 4: Merger

$\pi = P_1 q_1 - C_1(q_1, z) + P_2 q_2 - C_2(q_2, z)$

Approach 5: Taxation

$\pi_1 = P_1 q_1 - C_1(q_1, z) - tz$